

$A_4 \times \text{SU}(5)$ SUSY GUT of Flavour in 8d

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Abstract

We propose an $\text{SU}(5)$ SUSY GUT-Family model with A_4 family symmetry in 8d where the vacuum alignment is achieved in an elegant way by the use of boundary conditions on orbifolds. The model involves $\text{SU}(5)$ living in the 8d bulk, with matter living in 6d (or 4d) subspaces and Yukawa interactions occurring at a 4d point. The GUT group is broken to the Standard Model by the orbifold compactification, setting the GUT scale and leading to low energy supersymmetry and Higgs doublet-triplet splitting. The first two families of **10**-plets are doubled resulting in a lack of both desirable and unwanted GUT relations. The resulting four dimensional effective superpotential leads to a realistic description of quark and lepton masses and mixing angles including tri-bimaximal neutrino mixing and an inter-family mass hierarchy provided in part by volume suppression and in part by a Froggatt-Nielsen mechanism.

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1 Introduction

It is well known that the solar and atmospheric data are consistent with so-called tri-bimaximal (TB) mixing [1, 2],

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

The ansatz of TB lepton mixing matrix is interesting due to its symmetry properties which seem to call for a possibly discrete non-Abelian Family Symmetry in nature [3]. There has been a considerable amount of theoretical work in which the observed TB neutrino flavour symmetry may be related to some Family Symmetry [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. These models may be classified according to the way that TB mixing is achieved, namely either directly or indirectly [14]. The direct models are based on A_4 or S_4 , or a larger group that contains these groups as a subgroup, and in these models some of the generators of the Family Symmetry survive to form at least part of the neutrino flavour symmetry. In the indirect models, typically based on $\Delta(3n^2)$ or $\Delta(6n^2)$, none of the generators of the Family Symmetry appear in the neutrino flavour symmetry [14]. All of the above models rely on some kind of vacuum alignment mechanism, which in 4d SUSY models may arise from either F-terms in direct models or D-terms in indirect models (for a recent discussion see e.g. [15]).

The most ambitious models combine Family Symmetry with Grand Unified Theories (GUTs). The minimal Family Symmetry which contains triplet representations and can lead to TB mixing via the direct model approach is A_4 . The minimal simple GUT group is $SU(5)$. A direct model has been proposed which combines A_4 Family Symmetry with $SU(5)$ Supersymmetric (SUSY) GUTs [16]. This model was formulated in five dimensions (5d), in part to address the doublet-triplet splitting problem of GUTs [17], and in part to allow a viable description of the charged fermion mass hierarchies, by placing the lightest two ten-plets T_1, T_2 in the bulk, while the pentaplets F and T_3 are on the brane. It was subsequently shown how the geometry of 6d compactification may be used to generate the A_4 symmetry dynamically [18] (see also [19, 20]) and subsequently an $SU(5)$ SUSY GUT model in 6d was proposed in which the A_4 symmetry arises dynamically [21]. However, in both the 5d model [16] and the 6d model [21], the A_4 was broken at the effective 4d level in the standard way using F-terms to align the two flavons φ_T, φ_S via the introduction of so-called driving fields.

In the framework of extra dimensional theories, an attractive alternative mechanism for vacuum alignment arises based on orbifolding [22]. The required alignment at the zero mode level is achieved by imposing non-trivial boundary conditions on the orbifold [22]. In order to achieve the desired vacuum alignment for the two flavons φ_T, φ_S in an A_4 model it was demonstrated that it is necessary to formulate the model in 8

dimensions, which allows 4 compact dimensions which may be regarded as two complex compact dimensions z_1, z_2 , where each is subject to a particular 2d orbifold which gives the vacuum alignment for the particular flavon φ_T, φ_S living in those dimensions [22]. This mechanism has also been applied to S_4 where it has been shown that some of the desired alignments may be achieved in 6d [23]. However, in the case of A_4 it is clear that it is necessary to formulate the model in 8d in order to achieve successful vacuum alignments via boundary conditions, although so far only an illustrative model along these lines has been presented [22].

The purpose of this paper is to formulate the first realistic SU(5) SUSY GUT model with A_4 family symmetry in 8d where the vacuum alignment is straightforwardly achieved by the use of boundary conditions on orbifolds of the four compact dimensions. We emphasise that we are motivated to consider an 8d theory by the desire to achieve vacuum alignment in an elegant way using orbifold boundary conditions. It is not possible to implement this idea with lower dimensional models such as the 5d model in [16] or the 6d model in [21] since the desired alignment mechanism is not possible under a single orbifolding due to the requirement that the two triplet flavons φ_T and φ_S have different boundary conditions in order to have the different alignments at the zero mode level. Working in 8d also brings additional benefits, for example the inter-family mass hierarchies will arise in part due to suppression factors arising from an asymmetric geometric dilution of the wavefunctions in the four compact dimensions, although a $U(1)$ Froggatt-Nielsen family symmetry will also be required. In the 8d model the 4 extra dimensions are compactified onto 2 complex directions which are each orbifolded with \mathbb{Z}_2 and \mathbb{Z}_3 symmetries. These orbifoldings are also used to specify non-trivial boundary conditions on the various multiplets which break the SU(5) gauge symmetry and the extended $\mathcal{N} = 4$ symmetry to leave an effective $\mathcal{N} = 1$ Standard Model theory in 4 dimensions. It is worth noting that due to the orbifoldings the first two families of **10**-plets are duplicated introducing new GUT scale mass particles to the theory, although such a feature removes any desirable GUT predictions it also removes some unwanted GUT mass relations.

The layout of the remainder of the paper is as follows. In Section 2 we introduce the model and show how the 8 dimensions are compactified upon two $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}_3)$ orbifolds leading to gauge and SUSY breaking as above. We specify the superfield content and symmetries of the model. We describe the transformation of the fields under these orbifoldings which leads to an effective 4d Standard Model theory from the 8d SU(5) theory. We first show how the GUT group is broken and how this naturally leads to doublet-triplet splitting of the Higgs multiplets. We then discuss vacuum alignment in the 8d theory, and show how boundary conditions can lead to the desired alignment directions. We also discuss the values of the Higgs and flavon VEVs, including the effects of bulk suppression factors. In Section 3 we write down the effective 4d superpotential and the resulting mass matrices. We also analyse contributions from terms beyond the leading order to the mass matrices. Section 4 concludes the paper.

2 The Model

We are considering a model in 8 dimensions with the extra dimensions compactified on two 2d orbifolds as described in sec. 2.1. The $SU(5)$ gauge group lives in the full 8d bulk, with the 8d space compactified to 4d Minkowski space \times 4d compact dimensions with the two complex compact dimensions described by the coordinates z_1 and z_2 . We suppose that the 8d space is compactified by orbifolding. In the z_1 direction the \mathbb{Z}_2 orbifolding breaks the gauge symmetry and gives the alignment of the A_4 flavon φ_S , while the \mathbb{Z}_3 orbifold breaks the extended supersymmetry as described below. In the z_2 direction the \mathbb{Z}_2 orbifolding also breaks the gauge symmetry to the Standard Model in exactly the same way as in the z_1 direction, while the \mathbb{Z}_3 symmetry is used to give the alignment of the A_4 flavon φ_T as described in sec. 2.5 and [22].

We suppose that some of the matter and Higgs fields do not feel the full 8d but are restricted to live in a 6d subspace of the full 8d theory. The second family of $\mathbf{10}$'s, T_2 , live in the z_1 direction along with both Higgs multiplets, H_5 and $H_{\bar{5}}$. The first family of $\mathbf{10}$'s, T_1 , is placed in the z_2 direction. Similarly, the flavons φ_S , ξ and θ'' live in the z_1 direction, with φ_T and θ in the z_2 direction. We confine the other matter fields to live in a 4d subspace, with the three families of $\bar{\mathbf{5}}$ matter, F , and the third family of $\mathbf{10}$'s, T_3 , along with the three families of right-handed neutrinos, N , located at the 4 dimensional fixed point $z_1 = z_2 = 0$, with the Yukawa couplings given by the overlap of the wavefunctions at this fixed point. The particle content of the model is summarised in table 1.

Superfield	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_T	φ_S	ξ	θ	θ''
SU(5)	1	$\bar{\mathbf{5}}$	10	10	10	5	$\bar{\mathbf{5}}$	1	1	1	1	1
SM	1	(d^c, l)	(u_1^c, q_1, e_1^c)	(u_2^c, q_2, e_2^c)	(u_3^c, q_3, e_3^c)	H_u	H_d	φ_T	φ_S	ξ	θ	θ''
A_4	3	3	$1''$	$1'$	1	1	$1'$	3	3	1	1	$1''$
$U(1)$	0	0	2	1	0	0	0	0	0	0	-1	-1
\mathbb{Z}_3	ω	ω	ω	ω	ω	ω	ω	1	ω	ω	1	1
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0
Location	$z_1 = z_2 = 0$	$z_1 = z_2 = 0$	$z_1 = 0$	$z_2 = 0$	$z_1 = z_2 = 0$	$z_2 = 0$	$z_2 = 0$	$z_1 = 0$	$z_2 = 0$	$z_2 = 0$	$z_1 = 0$	$z_2 = 0$

Table 1: The Particle content and symmetries of the model.

A schematic diagram of the model is shown in figure 1. As both the z_1 and z_2 directions have a \mathbb{Z}_2 orbifolding breaking the gauge symmetry, doublet-triplet splitting of the Higgs multiplets occurs. However this results in half the $\mathbf{10}$ -plet becoming heavy. To overcome this, an extra copy of $\mathbf{10}$'s must be included in both directions with opposite parity under the \mathbb{Z}_2 symmetry. This results in the complete matter content and also allows us to escape unwanted GUT mass relations. In addition to the unwanted GUT

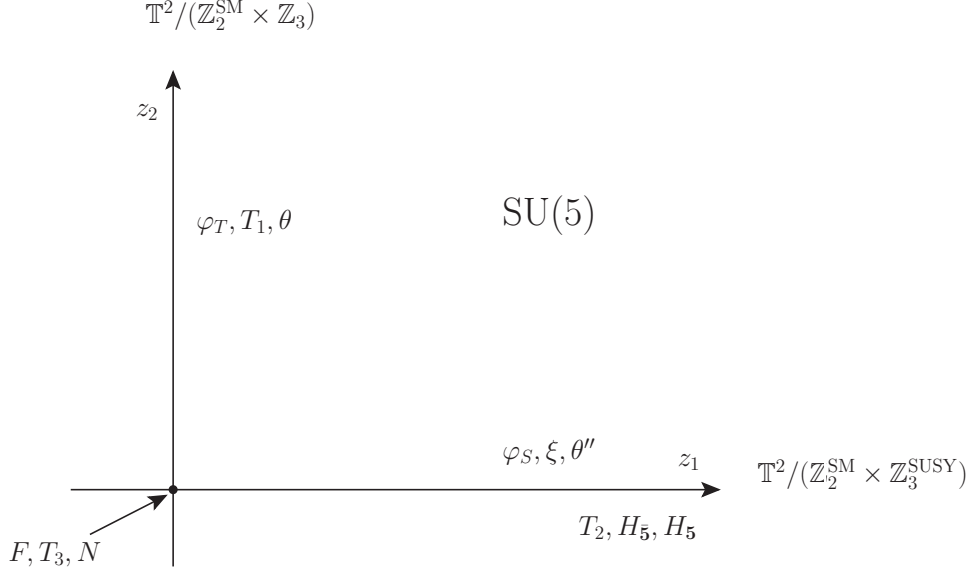


Figure 1: A Schematic diagram of the model. The $SU(5)$ gauge group is in the 8d bulk, represented here by the entire (z_1, z_2) plane, while matter and Higgs fields are confined to 6d subspaces, represented by the complex coordinate directions z_1 and z_2 , or to the 4d subspace, represented by the point at the origin. The First and Second families are placed in the z_2 and z_1 directions respectively. Because there is a gauge breaking orbifolding, in both directions, half of the **10**-plets become heavy so additional multiplets are introduced in both directions with opposite parity to obtain the full SM particle content.

mass relations the doubling of the first two families also prevents good GUT predictions such as the Gatto-Sartori-Tonin and Georgi-Jarlskog relations. The 8 dimensional theory has an A_4 family symmetry which is broken by three flavons φ_T, φ_S and ξ . The vacuum alignment of the flavons is achieved by imposing non-trivial boundary conditions on the flavons so that only the required alignment has a zero-mode. In addition to the A_4 flavour symmetry there is volume suppression for superpotential terms involving 6d fields. This suppression, however, turns out to be insufficient to account for realistic masses and mixings. To obtain a realistic pattern we also exploit the Froggatt-Nielsen mechanism [24] with a $U(1)$ symmetry and the two Froggatt-Nielsen flavons θ and θ'' living in the different orbifolded directions. We also make use of $U(1)_R$ and \mathbb{Z}_3 symmetries as shown in table 1.

2.1 The $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}_3)$ orbifolds

The orbifolding can be used to break both the gauge symmetry and SUSY [17]. Models have also been proposed [21] that combine these two ideas to give an extra dimensional GUT theory with a family symmetry arising from the compactification of the extra dimensions. In the present model we will not insist that the family symmetry is dynamically generated from the compactified geometry of extra dimensions, but merely suppose that it pre-exists in the 8d theory. However the part of the orbifold $\mathbb{T}^2/\mathbb{Z}_2$ described in this section is the same as that described in [18, 21] where the A_4 is dynamically generated. The new feature here is that we shall use orbifold boundary conditions to give the desired vacuum alignment for the flavons which break A_4 , thereby yielding TB neutrino mixing. We complexify the extra dimensions x_5, x_6 so that they are described by one complex coordinate $z_1 = x_5 + ix_6$. The extra dimensions are compactified on the a twisted torus defined by identifying the following translations:

$$z_1 \rightarrow z_1 + 1 \quad (2)$$

$$z_1 \rightarrow z_1 + \gamma \quad (3)$$

where $\gamma = e^{i\pi/3}$ and we have set $2\pi R_{z_1}$, the length of the extra dimension, to unity. We then impose the following identification:

$$\mathbb{Z}_2 : z_1 \rightarrow -z_1. \quad (4)$$

This defines the orbifold $\mathbb{T}^2/\mathbb{Z}_2$ as in [18, 21]. We can also impose a \mathbb{Z}_3 symmetry in order to define the orbifold $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}_3)$, we impose the following identification:

$$\mathbb{Z}_3 : z_1 \rightarrow \omega z_1. \quad (5)$$

Combining eqns. (2)-(5) gives the definition of the orbifold $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}_3)$ which is the complex direction denoted by z_1 in figure 1. We follow an analogous procedure for the remaining 2 extra dimensions by defining $z_2 = x_7 + ix_8$ and imposing the above definitions substituting z_2 for z_1 . In other words, we apply $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}_3)$ orbifolding separately in each of the z_1 and z_2 spaces. The overall orbifold has a single fixed point invariant under both the \mathbb{Z}_2 and \mathbb{Z}_3 transformations which is located at $z_{1,2} = 0$. It is at this 4d point that the Yukawa interactions occur.

2.2 SUSY Breaking

The full 8d theory is $\mathcal{N} = 1$ SU(5) and the 8d bulk of the theory contains the SU(5) gauge bosons. Because spinors in 8 dimensions contain a minimum of 16 real components then in 4 dimensions the effective theory must have $\mathcal{N} = 4$ supersymmetry [25]. In order to eliminate this extended supersymmetry we can impose boundary conditions on the multiplets so that they become heavy and play no part in the zero mode physics. The

$\mathcal{N} = 4$ vector multiplet decomposes into 3 chiral ϕ_i and one vector V $\mathcal{N} = 1$ multiplets. We can use the $\mathbb{T}^2/\mathbb{Z}_3$ part of the orbifolding to eliminate the unwanted multiplets by imposing the boundary conditions:

$$V(x^\mu, z_1, z_2) = V(x^\mu, \omega z_1, z_2) \quad (6)$$

$$\phi_i(x^\mu, z_1, z_2) = \omega \phi_i(x^\mu, \omega z_1, z_2), \quad (7)$$

where ω are the cube roots of unity, leaving $\phi = 0$ at the fixed point at $z_{1,2} = 0$. We are therefore left with an effective $\mathcal{N} = 1$ theory in 4 dimensions.

2.3 Gauge breaking through orbifolding

The breaking of the $SU(5)$ gauge group down to that of the Standard Model can be achieved by the \mathbb{Z}_2 part of the orbifolding. By using a single parity P_{SM} ,

$$P_{SM} = \begin{pmatrix} +1 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad (8)$$

we shall require that:

$$P_{SM} V_\mu(x, -z) P_{SM}^{-1} = +V_\mu(x, z). \quad (9)$$

Gauge boson fields of the standard model thus have positive parity and fields belonging to $SU(5)/G_{SM}$ have negative parity. Only fields with a positive parity have zero modes and therefore gauge bosons not belonging to the standard model gauge group become heavy and the gauge symmetry is broken. In our model both the z_1 and z_2 directions are orbifolded in this way, this allows us to relax unwanted GUT relations between the down quark and charged lepton mass matrices.

2.4 Higgs and doublet-triplet splitting

So far we have just considered the gauge sector of $SU(5)$. Adding the Higgs to the 6d theory is straightforward. In the $SU(5)$ GUT theory these are contained in the $\mathbf{5}$ -plet and $\mathbf{\bar{5}}$ -plet of Higgs fields. For the gauge breaking orbifold we choose:

$$\begin{aligned} P_{SM} H_{\mathbf{5}}(x, -z_1) &= +H_{\mathbf{5}}(x, z_1) \\ P_{SM} H_{\mathbf{\bar{5}}}(x, -z_1) &= +H_{\mathbf{\bar{5}}}(x, z_1) \end{aligned}$$

It is easy to see with the form of P_{SM} that the last three entries gain a minus sign which makes them heavy whereas the first two entries are left unchanged leaving them

light, resulting in a light doublet and a heavy coloured triplet. Similarly with the **10**-plets living in the z_1 and z_2 directions half the multiplet becomes heavy, however by introducing extra multiplets with opposite parity the full particle content is restored at zero mode. This feature also allows us to evade unwanted GUT relations.

2.5 Vacuum Alignment, VEVs and Expansion Parameters

In order to break the A_4 family symmetry we will impose non-trivial boundary conditions on flavons under the orbifoldings so that only a particular alignment survives to low energy. By imposing boundary conditions we are able to avoid introducing the driving fields and avoid having to write down a possibly complicated flavon potential. We will now describe the procedure for obtaining the alignment, closely following the procedure developed in [22] to which we refer the reader for more details. The first \mathbb{Z}_2 boundary condition,

$$\varphi_S(-z_1) = P_2 \varphi_S(z_1), \quad (10)$$

requires the matrix P_2 to be of order 2. For A_4 we have the elements in the fourth conjugacy class to choose from. We can choose the matrix $P_2 = S$ where S is given by

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (11)$$

in the basis of A_4 where S is diagonal. This makes it trivial to see which alignment is left as a zero mode. This choice leaves a single zero mode in the $(1, 0, 0)$ direction in this basis. To find what this alignment is in the T diagonal basis it is a simple matter to rotate the vector using (for example see [21]):

$$V = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix}, \quad (12)$$

This leaves us with the alignment $\varphi_S \propto (1, 1, 1)$ in the T diagonal basis. For the \mathbb{Z}_3 orbifolding we can impose the boundary condition

$$\varphi_T(\omega z_2) = P_3 \varphi_T(z_2) \quad (13)$$

and we can choose A_4 elements which have order 3. For P_3 we choose $P_3 = T$ where

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad (14)$$

This gives a single zero mode $\varphi_T \propto (1, 0, 0)$.

Turning to the VEVs themselves, for simplicity from now on we shall set the radii of the compact directions to $R_5 = R_6 = R_{z_1}$ and $R_7 = R_8 = R_{z_2}$, which implies that the Higgs VEVs are given by

$$\langle H_u(z_2) \rangle = \frac{v_u}{\sqrt{\pi^2 R_{z_1}^2 \sin \theta}}, \quad \langle H_d(z_1) \rangle = \frac{v_d}{\sqrt{\pi^2 R_{z_1}^2 \sin \theta}} \quad (15)$$

where we have included the effect of arbitrary twist angle θ on the torus [21]. For numerical estimates we will set the twist angle to 60° (by choosing $\gamma = e^{i\pi/3}$ in eqn. 3) as in [21] (although in the present model this is an arbitrary choice).

A useful feature of this setup is the suppression of the Yukawa couplings of fields living in the bulk. A field living in the 6d bulk of one of the orbifolded directions is related to its zero mode by

$$F(x^\mu, z) = \frac{1}{\sqrt{V}} F^0 + \dots \quad (16)$$

where the dots represent the higher, heavy modes and V is the volume of the extra dimensional space. The above expansion produces a factor s :

$$s = \frac{1}{\sqrt{V\Lambda^2}}. \quad (17)$$

This feature will produce suppression for couplings involving these bulk fields. Since we are considering 6 dimensional fields that live in either the z_1 or z_2 direction we will have two not necessarily equal volume factors, s_1 and s_2 :

$$s_1 = \frac{1}{\sqrt{\pi^2 R_{z_1}^2 \sin \theta \Lambda^2}} = \frac{1}{\sqrt{V_{z_1} \Lambda^2}} < 1 \quad (18)$$

and

$$s_2 = \frac{1}{\sqrt{\pi^2 R_{z_2}^2 \sin \theta \Lambda^2}} = \frac{1}{\sqrt{V_{z_2} \Lambda^2}} < 1. \quad (19)$$

Including volume suppression factors, we summarise the aligned flavon VEVs as follows,

$$\frac{\langle \varphi_T \rangle}{\Lambda} = \frac{1}{\sqrt{V_{z_2}}}(v_T, 0, 0), \quad (20)$$

$$\frac{\langle \varphi_S \rangle}{\Lambda} = \frac{1}{\sqrt{V_{z_1}}}(v_S, v_S, v_S), \quad (21)$$

$$\frac{\langle \xi \rangle}{\Lambda} = \frac{1}{V_{z_1}}u, \quad (22)$$

$$\frac{\langle \theta \rangle}{\Lambda} = \frac{1}{\sqrt{V_{z_2}}}t, \quad (23)$$

$$\frac{\langle \theta'' \rangle}{\Lambda} = \frac{1}{\sqrt{V_{z_1}}}t'' \quad (24)$$

We have defined the parameters v_T, v_S, t and t'' so that they are dimensionless recalling that 6d fields have mass dimension two. The Froggatt-Nielsen flavons θ, θ'' require no special vacuum alignment and are assumed to obtain VEVs t, t'' of $\mathcal{O}(1)$. Such VEVs can be obtained as in [16] by minimising the D-term scalar potential. Obtaining VEVs of $\mathcal{O}(1)$ can be found by assuming appropriate mass and coupling parameters.

3 Superpotentials and Mass Matrices

The couplings are localised at the single fixed point located at $z_1 = z_2 = 0$ in the extra dimensional space. The action reads

$$\int d^4x \int d^{(4)}z \int d^2\theta w(x) \delta(z_1) \delta(z_2) + h.c. = \int d^4x \int d^2\theta w(x) + h.c. \quad (25)$$

The effective superpotential w is expressed in terms of $\mathcal{N} = 1$ superfields can be decomposed into the following parts:

$$w = w_{\text{up}} + w_{\text{down}} + w_{\text{charged lepton}} + w_{\nu} + w_{\text{flavon}} \quad (26)$$

The fermion masses and mixings are given by the first three parts after $A_4, U(1)$ Froggatt-Nielsen and electroweak symmetry breaking. The w_{flavon} part concerns the flavon fields, however since the A_4 flavon alignment is given by the non-trivial boundary conditions imposed by the orbifolding we can avoid writing down explicitly the (possibly complicated) flavon potential. However without explicitly writing the flavon potential we do lose the ability to make specific claims on relations between the A_4 flavon VEVs.

3.1 Superpotentials

We shall now write down the superpotentials of the model (excluding w_ν which is discussed in sec. 3.3). We shall use Standard Model notation since the theory is broken to the Standard Model gauge group by the compactification. We have suppressed the coefficients in each term of the superpotentials and we would expect such coefficients to be of $\mathcal{O}(1)$. We shall use the notation for fields $(f)'$ where the field transforms as a $\mathbf{1}'$ and similarly $(f)''$ for a $\mathbf{1}''$ of A_4 .

$$\begin{aligned}
w_{\text{up}} \sim & \frac{1}{\Lambda} H_u q_3 u_3^c + \frac{\theta''}{\Lambda^4} H_u \{(q_2)' u_3^c + q_3 (u_2^c)'\} + \frac{\theta''^2}{\Lambda^7} H_u \{(q_2)' (u_2^c)'\} \\
& + \frac{\theta''^2}{\Lambda^6} H_u \{(q_1)'' u_3^c + q_3 (u_1^c)''\} + \frac{\theta''^3 + \theta^3}{\Lambda^9} H_u \{(q_2)' (u_1^c)'' + (q_1)'' (u_2^c)'\} \\
& + \frac{\theta''\theta^3 + \theta''^4}{\Lambda^{11}} H_u \{(q_1)'' (u_1^c)''\},
\end{aligned} \tag{27}$$

$$\begin{aligned}
w_{\text{down}} \sim & \frac{1}{\Lambda^3} (H_d)' (d^c \varphi_T)'' q_3 + \frac{\theta''}{\Lambda^6} (H_d)' (d^c \varphi_T)'' (q_2)' + \frac{\theta}{\Lambda^6} (H_d)' (d^c \varphi_T)' (q_2)' \\
& + \frac{\theta''^2}{\Lambda^8} (H_d)' (d^c \varphi_T)'' (q_1)'' \\
& + \frac{\theta''\theta}{\Lambda^8} (H_d)' (d^c \varphi_T)' (q_1)'' + \frac{\theta^2}{\Lambda^8} (H_d)' (d^c \varphi_T) (q_1)''
\end{aligned} \tag{28}$$

$$\begin{aligned}
w_{\text{charged lepton}} \sim & \frac{1}{\Lambda^3} (H_d)' (l \varphi_T)'' e_3^c + \frac{\theta''}{\Lambda^6} (H_d)' (l \varphi_T)'' (e^c_2)' + \frac{\theta}{\Lambda^6} (H_d)' (l \varphi_T)' (e^c_2)' \\
& + \frac{\theta''^2}{\Lambda^8} (H_d)' (l \varphi_T)'' (e^c_1)'' \\
& + \frac{\theta''\theta}{\Lambda^8} (H_d)' (l \varphi_T)' (e^c_1)'' + \frac{\theta^2}{\Lambda^8} (H_d)' (l \varphi_T) (e^c_1)''
\end{aligned} \tag{29}$$

3.2 Charged Fermion Mass Matrices

The Higgs multiplets obtain their VEVs along with the A_4 and $U(1)$ flavons $\varphi_T, \theta'', \theta$ as in Eqs.20-24 leading to mass matrices of the following form:

$$m_u \sim \begin{pmatrix} (s_1 s_2^3 t'' t^3 + s_1^4 t''^4) s_2^2 & (s_1^3 t''^3 + s_2^3 t^3) s_1 s_2 & s_1^2 t''^2 s_2 \\ (s_1^3 t''^3 + s_2^3 t^3) s_1 s_2 & s_1^2 t''^2 s_1^2 & s_1 t'' s_1 \\ s_1^2 t''^2 s_2 & s_1 t'' s_1 & 1 \end{pmatrix} s_1 v_u, \tag{30}$$

$$m_d \sim \begin{pmatrix} s_2^3 t^2 & s_2^2 s_1 t'' t & s_2 s_1^2 t''^2 \\ \dots & s_1 s_2 t & s_1^2 t'' \\ \dots & \dots & 1 \end{pmatrix} s_1 s_2 v_T v_d, \quad (31)$$

$$m_e \sim \begin{pmatrix} s_2^3 t^2 & \dots & \dots \\ s_2^2 s_1 t'' t & s_2 s_1 t & \dots \\ s_2 s_1^2 t''^2 & s_1^2 t'' & 1 \end{pmatrix} s_1 v_T v_d, \quad (32)$$

The dots in m_d and m_e are from higher order corrections to the vev of the φ_T flavon alignment. Such corrections come from the heavier modes which have a higher mass through orbifolding and will alter the alignment of φ_T as discussed in sec. 2.5.

We set $s_1 = \lambda$ and $s_2 = \lambda^{3/2}$ with $\lambda = 0.22$, we choose for simplicity $t = t'' = \mathcal{O}(1)$. We should make clear that taking $t = t'' = \mathcal{O}(1)$ means that we are not using the Froggatt-Nielsen mechanism to provide the suppression. Instead the hierarchies originate from the bulk suppression factors s_i . The mass matrices are then given by:

$$m_u \sim \begin{pmatrix} \lambda^7 & \lambda^{5.5} & \lambda^{3.5} \\ \lambda^{5.5} & \lambda^4 & \lambda^2 \\ \lambda^{3.5} & \lambda^2 & 1 \end{pmatrix} \lambda v_u. \quad (33)$$

The down sector matrix is given by,

$$m_d \sim \begin{pmatrix} \lambda^{4.5} & \lambda^4 & \lambda^{3.5} \\ \dots & \lambda^{2.5} & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} \lambda^{2.5} v_T v_d, \quad (34)$$

where again the dots represent contributions from the corrections to the vacuum alignment. The charged lepton mass matrix is given by

$$m_{\text{charged lepton}} \sim \begin{pmatrix} \lambda^{4.5} & \dots & \dots \\ \lambda^4 & \lambda^{2.5} & \dots \\ \lambda^{3.5} & \lambda^2 & 1 \end{pmatrix} \lambda^{2.5} v_T v_d, \quad (35)$$

In this model since the first two families are doubled, because the gauge breaking orbifolding makes half of the **10**-plets heavy the, GUT relation $m_{\text{down}} = m_{\text{charged lepton}}^T$ for the first two families is not valid.

These mass matrices give us approximate quark masses and mixing angles of the correct order of magnitude. For example the quark mixing angles are given roughly by,

$$\theta_{12} = \mathcal{O}(\lambda^{1.5}) \quad (36)$$

$$\theta_{23} = \mathcal{O}(\lambda^2) \quad (37)$$

$$\theta_{13} = \mathcal{O}(\lambda^{3.5}). \quad (38)$$

So far we have not specified the size of v_T and v_S , However from the ratio of the top and bottom quark masses we expect

$$\begin{aligned} \frac{m_b}{m_t} &= \lambda^{3/2} \frac{v_d}{v_u} v_T \sim \lambda^2 \\ \Rightarrow v_T &\sim \frac{\lambda^{1/2}}{\tan \beta} \sim \frac{1}{2 \tan \beta} \end{aligned} \quad (39)$$

where $\frac{v_d}{v_u} = \tan \beta$.

3.3 Neutrino sector

In the neutrino sector the right-handed neutrino A_4 triplets live at the fixed point. The φ_S lives in the z_1 direction along with the A_4 singlet flavon ξ . After these flavons develop a vev the gauge singlets N become heavy and the seesaw mechanism takes place similar to [16],[21] with the alteration that a zero vev A_4 singlet flavon is no longer required as the vacuum alignment is determined by boundary conditions rather than by the use of driving fields. Thus we have,

$$w_\nu \sim \frac{y^D}{\Lambda} H_u(Nl) + \frac{1}{\Lambda} x_a \xi(NN) + \frac{x_b}{\Lambda} \varphi_S(NN) \quad (40)$$

After the fields develop VEVs, the gauge singlets N become heavy and the see-saw mechanism takes place as discussed in detail in [4], leading to the effective mass matrix for the light neutrinos:

$$m_\nu \sim \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s_1(v_u)^2}{\Lambda} \quad (41)$$

where

$$a \equiv \frac{2x_a s_1 u}{(y^D)^2}, b \equiv \frac{2x_b s_1 v_S}{(y^D)^2}.$$

The neutrino mass matrix is diagonalised by the transformation

$$U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$$

with U_ν given by:

$$U_\nu = \begin{pmatrix} -\sqrt{2/3} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

which is of the TB form in Eq. (1). However, although we have TB neutrino mixing in this model we do not have exact TB lepton mixing due to fact that the charged lepton mass matrix is not diagonal in this basis. Thus there will be charged lepton mixing corrections to TB mixing resulting in mixing sum rules as discussed in [5, 26].

3.4 Higher order corrections

We will now discuss corrections to the mass matrices, such corrections come from additional flavon insertion of $\varphi_T, \varphi_S, \xi$ and θ, θ'' , and also from corrections to the vacuum alignment of the A_4 triplet flavons φ_T and φ_S .

3.4.1 corrections to m_{up}

The leading order terms in the up sector are of the form $\theta^m \theta'^n H_u q_i u_j$. Terms are gauge and A_4 singlets, to create higher order terms we need to introduce flavon fields. The most straightforward way to do this is to introduce two flavon fields $(\varphi_T \varphi_T)_1$, since φ_T is an A_4 triplet we need the two triplet fields in order to construct an A_4 singlet. Such terms will lead to entries in the mass matrix suppressed by a factor of $s_2^2 v_T^2$. Due to the \mathbb{Z}_3 symmetry the flavon fields $\varphi_S, \xi, \tilde{\xi}$ must enter at the three flavon level so entries will be suppressed by a factor of $s_1^3 v_S^2 u, s_1^3 v_S^3$ and $s_1^3 u^3$ relative to the leading order term. Using the values assumed in sec. 3.2 the corrections enter at $\mathcal{O}(\lambda^3)$ relative to the leading order term.

3.4.2 corrections to m_d and m_e

In the down quark mass matrix sub-leading corrections fill in the entries indicated by dots in Eq. 31. Entries in the matrix are generated by terms of the form $\theta^m \theta'^n H'_d ((d^c \varphi_T) q_i + (l \varphi_T) e_i^c)$, higher order terms can come from replacing φ_T with a product of flavon fields or including the effect of the corrections to the VEV of φ_T .

The obvious substitution is to replace φ_T with $\varphi_T \varphi_T$, this is compatible with the \mathbb{Z}_3 charges and results in corrections with the same form as m_{down} but with an extra overall suppression of $s_2 v_T$. Using the values assumed in sec. 3.2 this type of correction enters at the level of $\mathcal{O}(\lambda^{3/2})$

If we include the corrections to the alignment of the VEV of φ_T then we fill in the entries indicated by dots in Eq. (31). Such corrections originate from higher, heavy modes of the flavon field φ_T , such corrections would be suppressed by an order of s_2 relative to the leading order term giving corrections to the mass matrix of the form:

$$\delta m_{\text{down}} \sim \begin{pmatrix} \lambda^5 & \lambda^5 & \lambda^5 \\ \lambda^{3.5} & \lambda^{3.5} & \lambda^{3.5} \\ \lambda^{1.5} & \lambda^{1.5} & \lambda^{1.5} \end{pmatrix} \lambda^{2.5} v_T v_d, \quad (42)$$

i.e. the corrections are suppressed by $\mathcal{O}(\lambda^{3/2})$ relative to the largest term in each row (or column for $m_{\text{charged lepton}}$).

As remarked, since the first two families are doubled, because the gauge breaking orbifolding makes half of the **10**-plets heavy the, GUT relation $m_{\text{down}} = m_{\text{charged lepton}}^T$ for the first two families is not valid. It does however hold up to orders of magnitude

for the individual families so that the power of λ is the same for each family though the (suppressed) $\mathcal{O}(1)$ coefficient can be different for each family.

3.4.3 corrections to m_ν

The leading order Dirac mass term for the neutrinos is $H_u(Nl)$, sub-leading corrections to this term enter with a single flavon insertion of φ_T so the resulting term is $H_u(\varphi_T Nl)$ this results in the sub-leading corrections entering at the $s_2 v_T$ level. Using the values assumed in sec. 3.2 the corrections enter at the $\mathcal{O}(\lambda^{3/2})$ level.

Corrections to the Majorana mass matrix can arise from a number of terms. This is due to the term (NN) being a product of two triplets and can thus be a triplet or any of the singlet representations of A_4 . Corrections to the Majorana mass matrix can have one extra flavon insertion relative to the leading order terms $\xi(NN), (\varphi_S NN)$. For example the term $(\varphi_S \varphi_T)(NN)$ is allowed by the \mathbb{Z}_3 symmetry and leads to corrections of order $s_2 v_T$. After the seesaw mechanism takes place corrections to the neutrino masses and tri-bimaximal mixing are of order $s_2 v_T$. Using the values assumed in sec. 3.2 these corrections are $\mathcal{O}(\lambda^{3/2})$ relative to the leading order term.

4 Conclusion

We have proposed the first realistic $\mathcal{N} = 1$ SUSY SU(5) GUT model in 8 dimensions with an A_4 family symmetry where the vacuum alignment is straightforwardly achieved by the use of boundary conditions on orbifolds of the four compact dimensions. The low energy theory is the usual $\mathcal{N} = 1$ SUSY Standard Model in 4 dimensions but with predictions for quark and lepton (including neutrino) masses and mixing angles. For example, the low energy 4d model naturally has TB mixing at the first approximation and reproduces the correct mass hierarchies for quarks and charged leptons and the CKM mixing pattern. The presence of SU(5) GUTs means that the charged lepton mixing angles are non-zero resulting in predictions such as lepton mixing sum rules.

We were motivated to consider an 8d theory by the desire to achieve the A_4 flavon vacuum alignment in an elegant way using orbifold boundary conditions. Such boundary conditions result in the required alignment surviving at the zero mode level, and in relatively small corrections to the alignment resulting from heavy higher modes. However the extra dimensional set up also provides familiar added benefits such as orbifold gauge and SUSY breaking with doublet-triplet splitting of the $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs multiplets, making the coloured triplets heavy. Because the first two generations of $\mathbf{10}$ -plets are doubled, both unwanted and desirable GUT relations are also avoided. The lack of such relations introduces more freedom into the theory. The specific model in in table 1 and figure 1 also includes a Froggatt-Nielsen $U(1)$ symmetry, which, together with the bulk suppression factors, leads to the desired inter-family hierarchies.

Finally we comment on the possible relation between the 8d orbifold GUT-Family model considered here and string theory. At first glance there is an intriguing similarity between the model here and the F-theory GUT recently discussed [27]. In both cases the $SU(5)$ GUT gauge group lives in the full 8d space, and also the matter and Higgs fields lie on matter curves in a 6d subspace, corresponding to two extra complex dimensions $z_{1,2}$, with Yukawa couplings occurring at a 4d point [27]. However any possible connection would be more subtle than this, since firstly one must uplift the 8d orbifold GUT-Family model here into full heterotic string theory, then one must identify duality relations between the heterotic string theory and F-theory as discussed in [28]. Nevertheless the 8d orbifold GUT-Family model here may provide a useful stepping-stone towards some future unified string theory (including gravity, albeit perhaps decoupled in some limit) in which GUT breaking and the emergence of family symmetry, spontaneously broken with a particular vacuum alignment, can be naturally explained as the result of the compactification of extra dimensions.

Acknowledgements

SFK acknowledges the support of a Royal Society Leverhulme Trust Senior Research Fellowship and the STFC Rolling Grant ST/G000557/1.

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